

PROOF OF FORMULA 3.312.1

$$\int_0^{\infty} (1 - e^{-x/b})^{\nu-1} e^{-\mu x} dx = bB(b\mu, \nu)$$

Let $t = e^{-x/b}$ to obtain

$$\int_0^{\infty} (1 - e^{-x/b})^{\nu-1} e^{-\mu x} dx = b \int_0^1 (1 - t)^{\nu-1} t^{b\mu-1} dt.$$

The integral representation

$$B(u, v) = \int_0^1 t^{u-1} (1 - t)^{v-1} dt,$$

gives the result.