PROOF OF FORMULA 3.317.1

$$\int_{-\infty}^{\infty} \left[\frac{1}{1+e^{-x}} - \frac{1}{(1+e^{-x})^{\mu}} \right] dx = \psi(\mu) + \gamma$$

The change of variables $t = e^{-x}$ gives

$$\int_{-\infty}^{\infty} \left[\frac{1}{1+e^{-x}} - \frac{1}{(1+e^{-x})^{\mu}} \right] dx = \int_{0}^{\infty} \left[\frac{1}{1+t} - \frac{1}{(1+t)^{\mu}} \right] \frac{dt}{t}.$$

The integral representation 8.361.2:

$$\psi(a) = \int_{0}^{\infty} \left[e^{-t} - \frac{1}{(1+t)^{a}} \right] \frac{dt}{t}$$

for the digamma function ψ, combined with the special case $\nu = 1$ and the value $\psi(1) = -\gamma$ give the result given here.