

PROOF OF FORMULA 3.323.1

$$\int_1^{\infty} e^{-qx-x^2} dx = \frac{\sqrt{\pi}}{2} e^{q^2/4} \left[1 - \operatorname{erf} \left(1 + \frac{q}{2} \right) \right]$$

Complete the square to obtain

$$-x^2 - qx = -(x + q/2)^2 + \frac{q^2}{4}.$$

The change of variables $t = x + q/2$ gives

$$\int_1^{\infty} e^{-qx-x^2} dx = e^{q^2/4} \int_{1+q/2}^{\infty} e^{-t^2} dt.$$

The result now follows from the representation

$$1 - \operatorname{erf}(a) = \frac{2}{\sqrt{\pi}} \int_a^{\infty} e^{-t^2} dt.$$