

PROOF OF FORMULA 3.333.1

$$\int_{-\infty}^{\infty} \frac{e^{-sx} dx}{\exp(e^{-x}) - 1} = \Gamma(s)\zeta(s)$$

Let $t = e^{-x}$ to obtain

$$\int_{-\infty}^{\infty} \frac{e^{-sx} dx}{\exp(e^{-x}) - 1} = \int_0^{\infty} \frac{t^{s-1} dt}{e^t - 1}.$$

This integral is evaluated in 3.411.1 and it has value $\Gamma(s)\zeta(s)$.