PROOF OF FORMULA 3.342

$$\int_0^1 e^{-px\ln x}\,dx = \int_0^1 x^{-px}\,dx = \sum_{k=1}^{\infty} \frac{p^{k-1}}{k^k}$$

Expand the exponential to get

$$\int_0^1 e^{-px\ln x}\,dx = \sum_{k=0}^{\infty} \frac{(-1)^k p^k}{k!} \int_0^1 x^k \ln^k x\,dx.$$

The change of variables $x = e^{-t}$ gives

$$\int_0^1 x^k \ln^k x\,dx = (-1)^k \int_0^{\infty} t^k e^{-(k+1)t}\,dt.$$

The result now follows from making $s = (k+1)t$.