

PROOF OF FORMULA 3.361.3

$$\int_{-1}^{\infty} \frac{e^{-qx}}{\sqrt{1+x}} dx = e^q \sqrt{\frac{\pi}{q}}$$

Let $t = x + 1$ to obtain

$$\int_{-1}^{\infty} \frac{e^{-qx}}{\sqrt{x}} dx = e^q \int_0^{\infty} \frac{e^{-qt}}{\sqrt{t}} dt.$$

The change of variables $t = v/\sqrt{q}$ gives

$$\int_0^{\infty} \frac{e^{-qt}}{\sqrt{t}} dt = \frac{2}{\sqrt{q}} \int_0^{\infty} e^{-v^2} dv.$$

The result now follows from

$$\int_0^{\infty} e^{-v^2} dv = \frac{\sqrt{\pi}}{2}.$$