

PROOF OF FORMULA 3.363.1

$$\int_a^\infty \frac{\sqrt{x-a}}{x} e^{-\mu x} dx = \sqrt{\frac{\pi}{\mu}} e^{-a\mu} - \pi\sqrt{a} (1 - \operatorname{erf}(\sqrt{a\mu}))$$

Let $x = a + t^2$ to obtain

$$\int_a^\infty \frac{\sqrt{x-a}}{x} e^{-\mu x} dx = 2e^{-a\mu} J,$$

with

$$J = \int_0^\infty \frac{t^2 e^{-\mu t^2}}{a + t^2} dt = \int_0^\infty e^{-\mu t^2} dt - a \int_0^\infty \frac{e^{-\mu t^2}}{a + t^2} dt$$

and this gives $J = \sqrt{\pi}/2\sqrt{\mu} - aK$, where

$$K = \int_0^\infty \frac{e^{-\mu t^2}}{a + t^2} dt.$$

Let $t = r/\sqrt{\mu}$ to obtain

$$\int_a^\infty \frac{\sqrt{x-a}}{x} e^{-\mu x} dx = \sqrt{\frac{\pi}{\mu}} e^{-a\mu} - 2a\sqrt{a} e^{-a\mu} \int_0^\infty \frac{e^{-r^2}}{r^2 + a\mu} dr.$$

The change of variables $s = r^2 + a\mu$ gives

$$\int_a^\infty \frac{\sqrt{x-a}}{x} e^{-\mu x} dx = \sqrt{\frac{\pi}{\mu}} e^{-a\mu} - a\sqrt{\mu} \int_b^\infty \frac{e^{-s} ds}{s\sqrt{s-b}},$$

with $b = a\mu$. Let $s = b\tau$ to obtain

$$\int_b^\infty \frac{e^{-s} ds}{s\sqrt{s-b}} = \frac{1}{\sqrt{b}} \int_1^\infty \frac{e^{-b\tau} d\tau}{\tau\sqrt{\tau-1}}$$

The integral obtained by differentiating with respect to b is in 3.362.1. Integrating back yields the result.