PROOF OF FORMULA 3.364.1

$$\int_0^2 \frac{e^{-px}\,dx}{\sqrt{x(2-x)}} = \pi e^{-p} I_0(p)$$

The integral representation of the *imaginary Bessel function* is

$$I_\nu(z) = \frac{z^\nu}{\Gamma(\nu+\frac{1}{2})\Gamma(\frac{1}{2})2^\nu} \int_{-1}^1 (1-t^2)^{\nu-1/2} e^{\pm zt}\,dt.$$

The chnage of variables $t = x - 1$ yields

$$\int_0^2 \frac{e^{-px}\,dx}{\sqrt{x(2-x)}} = e^{-p} \int_{-1}^1 \frac{e^{-pt}\,dt}{\sqrt{1-t^2}}.$$

The choice $\nu = 0$ and $z = p$ gives the result.