PROOF OF FORMULA 3.365.2

$$\int_u^\infty \frac{xe^{-xt}\,dx}{\sqrt{x^2-u^2}} = uK_1(tu)$$

The integral representation for the Bessel function K_ν appears in 8.432.3:

$$K_\nu(z) = \frac{z^\nu\sqrt{\pi}}{2^\nu\Gamma(\nu+1/2)} \int_1^\infty e^{-tz}(t^2-1)^{\nu-1/2}.$$

In particular

$$K_0(z) = \int_1^\infty \frac{e^{-zt}\,dt}{\sqrt{t^2-1}} \text{ and } K_1(z) = z\int_1^\infty e^{-zt}\sqrt{t^2-1}\,dt.$$

The change of variables $x = tu$ gives

$$K_0(z) = \int_u^\infty \frac{e^{-zx/u}\,dx}{\sqrt{x^2-u^2}}.$$

Therefore

$$\int_u^\infty \frac{xe^{-xz}\,dx}{\sqrt{x^2-u^2}} = uK_0'(uz).$$

The relation $K_0'(z) = -K_1(z)$ can be established by integration by parts the identity

$$K_0'(z) = -\int_1^\infty \frac{te^{-zt}\,dt}{\sqrt{t^2-1}}.$$

This gives the result.