

PROOF OF FORMULA 3.369

$$\int_0^{\infty} \frac{e^{-\mu x} dx}{(x+a)^{3/2}} = \frac{2}{\sqrt{a}} - 2\sqrt{\pi\mu}e^{a\mu} (1 - \operatorname{erf}(\sqrt{a\mu}))$$

The change of variables $t = (x+a)/\mu$ yields

$$\int_0^{\infty} \frac{e^{-\mu x} dx}{(x+a)^{3/2}} = \sqrt{\mu}e^{\mu a} \int_{\mu a}^{\infty} t^{-3/2}e^{-t} dt.$$

Integration by parts, writing $b = \mu a$ and making the change of variables $t = y^2$ produces

$$\int_{\mu a}^{\infty} t^{-3/2}e^{-t} dt = \frac{2e^{-b}}{\sqrt{b}} - 4 \int_{\sqrt{b}}^{\infty} e^{-y^2} dy.$$

Replace and use

$$\operatorname{erf}(a) = \sqrt{\frac{\pi}{2}} \int_0^a e^{-y^2} dy$$

to obtain the result. The identity

$$\int_0^a e^{-y^2} dy + \int_a^{\infty} e^{-y^2} dy = \frac{\sqrt{\pi}}{2},$$

is useful in the simplification of the answer.