

**PROOF OF FORMULA 3.411.16**

$$\int_{-\infty}^{\infty} \frac{x^2 e^{-\mu x}}{1 + e^{-x}} dx = \frac{\pi^3 [2 - \sin^2(\pi\mu)]}{\sin^3(\pi\mu)}$$

The change of variables  $t = e^{-x}$  gives

$$\int_{-\infty}^{\infty} \frac{x^2 e^{-\mu x}}{1 + e^{-x}} dx = \int_0^{\infty} \frac{t^{\mu-1} \ln^2 t dt}{1 + t}.$$

The formula

$$\int_0^{\infty} \frac{t^{\mu-1} dt}{1 + t} = \frac{\pi}{\sin \pi\mu}$$

follows from the integral representation for the beta function

$$B(a, b) = \int_0^{\infty} \frac{t^{a-1} dt}{(1 + t)^{a+b}}$$

and the identity  $B(a, 1 - a) = \pi / \sin \pi a$ .

The original integral is obtained by differentiating twice with respect to the parameter  $\mu$ .