PROOF OF FORMULA 3.411.24

$$\int_{-\infty}^{\infty} \frac{xe^{\mu x}\,dx}{e^{\nu x}-1} = \left(\frac{\pi}{\nu \sin\frac{\pi\mu}{\nu}}\right)^2$$

Formula 3.313.1 states that

$$\int_{-\infty}^{\infty} \frac{e^{-\mu x}\,dx}{1-e^{-x}} = \pi \cot \pi\mu.$$

The change of variables $x = \nu t$ gives

$$\int_{-\infty}^{\infty} \frac{e^{(1-\mu)\nu t}\,dt}{e^{\nu t}-1} = \frac{\pi}{\nu} \cot \pi\mu.$$

This can be written as

$$\int_{-\infty}^{\infty} \frac{e^{\mu t}\,dt}{e^{\nu t}-1} = -\frac{\pi}{\nu} \cot \frac{\pi\mu}{\nu}.$$

The result follows by differentiating with respect to the parameter μ.