PROOF OF FORMULA 3.411.25

$$\int_0^\infty x\frac{1+e^{-x}}{e^x-1}\,dx = \frac{\pi^2}{3}-1$$

The change of variables $t = e^{-x}$ gives

$$\int_0^\infty x\frac{1+e^{-x}}{e^x-1}\,dx = -\int_0^1 \ln t\,\frac{1+t}{1-t}\,dt.$$

This integral is evaluated in 4.231.4 with value $\pi^2/3-1$.