

PROOF OF FORMULA 3.419.4

$$\int_{-\infty}^{\infty} \frac{x^3 dx}{(\beta + e^x)(1 - e^{-x})} = \frac{(\pi^2 + \ln^2 \beta)^2}{4(\beta + 1)}$$

In Part 1 we have established the formula for

$$h_n(a) = \int_0^{\infty} \frac{\ln^{n-1} t dt}{(t-1)(t+a)}$$

that states

$$\begin{aligned} n(1+a)h_n(a) &= (-1)^n n! [1 + (-1)^n] \zeta(n) \\ &+ \sum_{j=0}^{\lfloor n/2 \rfloor} \binom{n}{2j} (2^{2j} - 2) (-1)^{j-1} B_{2j} \pi^{2j} \ln^{n-2j} a. \end{aligned}$$

The change of variables $t = e^{-x}$ shows that

$$h_n(a) = \int_{-\infty}^{\infty} \frac{x^{n-1} dx}{(1 - e^{-x})(a + e^{-x})}.$$

The present entry corresponds to the value $n = 4$.