PROOF OF FORMULA 3.434.1

$$\int_0^\infty \frac{e^{-ax} - e^{-bx}}{x^{\rho+1}}\, dx = \frac{b^\rho - a^\rho}{\rho}\Gamma(1-\rho)$$

Start with

$$\int_0^\infty \frac{e^{-ax} - e^{-bx}}{x^{\rho+1}}\, dx = \int_0^\infty x^{-\rho-1}e^{-ax}\, dx - \int_0^\infty x^{-\rho-1}e^{-bx}\, dx.$$

Let $t = ax$ in the first integral and $t = bx$ in the secodn one to obtain the result.