PROOF OF FORMULA 3.452.1

$$\int_0^\infty \frac{x\,dx}{\sqrt{e^x-1}} = 2\pi \ln 2$$

The change of variables $t = e^{-x}$ gives

$$\int_0^\infty \frac{x\,dx}{\sqrt{e^x-1}} = -\int_0^1 \frac{\ln t\,dt}{\sqrt{t}\,\sqrt{1-t}}.$$

Then $t = u^2$ produces

$$\int_0^1 \frac{\ln t\,dt}{\sqrt{t}\,\sqrt{1-t}} = 4\int_0^1 \frac{\ln u\,du}{\sqrt{1-u^2}}.$$

This is evaluated in 4.241.7 to produce the result.