

PROOF OF FORMULA 3.457.1

$$\int_0^{\infty} x e^{-x} (1 - e^{-2x})^{n-1/2} dx = \frac{(2n-1)!! \pi}{4(2n)!!} (\gamma + \psi(n+1) + 2 \ln 2)$$

The change of variables $t = e^{-x}$ gives

$$\int_0^{\infty} x e^{-x} (1 - e^{-2x})^{n-1/2} dx = - \int_0^1 (1 - t^2)^{n-1/2} \ln t dt.$$

The result now follows from entry 4.241.5 which states that

$$\int_0^1 \sqrt{(1-x^2)^{2n-1}} \ln x dx = -\frac{(2n-1)!!}{(2n)!!} \frac{\pi}{4} \left(2 \ln 2 + \sum_{k=1}^n \frac{1}{k} \right).$$

This form is equivalent to the one stated here.