PROOF OF FORMULA 3.457.3

$$\int_{-\infty}^{\infty} \frac{x\,dx}{(a^2e^x + e^{-x})^\mu} = -\frac{1}{2a^\mu} B\left(\frac{\mu}{2}, \frac{\mu}{2}\right) \ln a$$

Let $t = ae^{-x}$ to obtain

$$\int_{-\infty}^{\infty} \frac{x\,dx}{(a^2e^x + e^{-x})^\mu} = \frac{1}{a^\mu} \int_0^\infty \frac{t^{\mu-1}\,(\ln t - \ln a)\,dt}{(1+t^2)^\mu}.$$

The change of variables $s = t^2$ gives

$$\int_{-\infty}^{\infty} \frac{x\,dx}{(a^2e^x + e^{-x})^\mu} = \frac{1}{2a^\mu} \int_0^\infty \frac{s^{\mu/2-1}\ln s\,ds}{(1+s)^\mu} - \frac{\ln a}{2a^\mu} \int_0^\infty \frac{s^{\mu/2-1}\,ds}{(1+s)^\mu}.$$

The first intel vanishes: this follows directly from the change of variables $s \mapsto 1/s$. The second integral is evaluated as $B(\mu/2, \mu/2)$ using the representation

$$B(p,q) = \int_0^\infty \frac{s^{p-1}\,ds}{(1+s)^{p+q}}.$$