FORMULA 3.462.5

$$\int_0^\infty xe^{-\mu x^2 - 2\nu x}\, dx = \frac{1}{2\mu} - \frac{\nu}{2\mu}\sqrt{\frac{\pi}{\mu}} e^{\nu^2/\mu} \left[1 - \text{erf}\left(\frac{\nu}{\sqrt{\mu}} \right) \right]$$

Let $t = \sqrt{\mu}x$ to obtain

$$\int_0^\infty xe^{-\mu x^2 - 2\nu x}\, dx = \frac{1}{\mu} \int_0^\infty te^{-t^2 - 2\alpha t}\, dt$$

with $\alpha = \nu/\sqrt{\mu}$. The change of variable $s = t + \alpha$ gives

$$\int_0^\infty xe^{-\mu x^2 - 2\nu x}\, dx = \frac{1}{\mu} e^{\alpha^2} \int_\alpha^\infty (s - \alpha)e^{-s^2} ds.$$

Then

$$\int_\alpha^\infty (s - \alpha)e^{-s^2} ds = \int_\alpha^\infty se^{-s^2} ds - \alpha \int_\alpha^\infty e^{-s^2} ds.$$

The first integral is $\frac{1}{2}e^{-\alpha^2}$ and the second one is written as

$$\int_\alpha^\infty e^{-s^2} ds = \frac{\sqrt{\pi}}{2} - \int_0^\alpha e^{-s^2} ds = \frac{\sqrt{\pi}}{2} \left(1 - \text{erf}(\alpha) \right).$$