

**PROOF OF FORMULA 3.462.6**

$$\int_{-\infty}^{\infty} x e^{-px^2+2qx} dx = \frac{q}{p} \sqrt{\frac{\pi}{p}} e^{q^2/p}$$

Complete the square gives

$$-px^2 + 2qx = -p \left[ (x - q/p)^2 - q^2/p \right].$$

The change of variables  $t = x - q/p$  gives

$$\int_{-\infty}^{\infty} x e^{-px^2+2qx} dx = e^{q^2/p} \int_{-\infty}^{\infty} (t + q/p) e^{-pt^2} dt.$$

The integral

$$\int_{-\infty}^{\infty} t e^{-pt^2} dt = 0$$

by symmetry. The second one can be scaled to get

$$\int_{-\infty}^{\infty} e^{-pt^2} dt = \frac{1}{\sqrt{p}} \int_{-\infty}^{\infty} e^{-y^2} dy = \sqrt{\frac{\pi}{p}}.$$

This is the result.