PROOF OF FORMULA 3.462.8

$$\int_{-\infty}^{\infty} x^2 e^{-\mu x^2+2\nu x}\,dx = \frac{e^{\nu^2/\mu}}{2\mu}\sqrt{\frac{\pi}{\mu}}\left(1+\frac{2\nu^2}{\mu}\right)$$

Let $t=\sqrt{\mu}x$ to obtain

$$\int_{-\infty}^{\infty} x^2 e^{-\mu x^2+2\nu x}\,dx = \frac{1}{\mu\sqrt{\mu}}\int_{-\infty}^{\infty} t^2 e^{-t^2+2\alpha t}\,dt$$

where $\alpha=\nu/\sqrt{\mu}$. This can be written as

$$\int_{-\infty}^{\infty} t^2 e^{-t^2+2\alpha t}\,dt = e^{\alpha^2}\int_{-\infty}^{\infty}(s+\alpha)^2 e^{-s^2}\,ds.$$

Expanding the binomial reduces the problem to the calculation of three integrals:

$$\int_{-\infty}^{\infty} s^2 e^{-s^2}\,ds = \frac{\sqrt{\pi}}{2}$$

that can be evaluated via the change of variables $s=\sqrt{u}$ and recognizing the resulting integral as $\Gamma(3/2)$;

$$\int_{-\infty}^{\infty} s e^{-s^2}\,ds = 0,$$

by symmetry and

$$\int_{-\infty}^{\infty} e^{-s^2}\,ds = \sqrt{\pi}.$$