

**PROOF OF FORMULA 3.466.1**

$$\int_0^\infty \frac{e^{-\mu^2 x^2} dx}{x^2 + b^2} = (1 - \operatorname{erf}(b\mu)) \frac{\pi}{2b} e^{b^2 \mu^2}$$

Let  $x = bt$  and replace  $b\mu$  by  $a$  to convert the formula into

$$\int_0^\infty \frac{e^{-a^2 t^2} dx}{1 + t^2} = (1 - \operatorname{erf}(a)) \frac{\pi}{2} e^{a^2}.$$

Define

$$f(a) := \int_0^\infty \frac{e^{-a^2 t^2} dx}{1 + t^2},$$

and observe that

$$f'(a) = -2a \int_0^\infty e^{-a^2(t^2+1)} dt = -\sqrt{\pi} e^{-a^2}.$$

The initial value  $f(0) = \pi/2$  and the definition

$$\operatorname{erf}(s) = \frac{2}{\sqrt{\pi}} \int_0^s e^{-t^2} dt,$$

give the result.