PROOF OF FORMULA 3.471.14

$$\int_0^1 \frac{\exp\left(1 - \frac{1}{x}\right) - x^\nu}{x(1 - x)}\, dx = \psi(\nu)$$

Let $t = 1/x - 1$ to obtain

$$\int_0^1 \frac{\exp\left(1 - \frac{1}{x}\right) - x^\nu}{x(1 - x)}\, dx = \int_0^\infty \left[e^{-t} - (1 + t)^{-\nu}\right] \frac{dt}{t}.$$

This is one of the fundamental integral representations of ψ. It appears as 8.361.2.