PROOF OF FORMULA 3.471.3

$$\int_0^a x^{-\mu-1}(a-x)^{\mu-1}e^{-b/x}\,dx = b^{-\mu}a^{\mu-1}\Gamma(\mu)e^{-b/a}$$

Let $t = b/x$ to obtain

$$\int_0^a x^{-\mu-1}(a-x)^{\mu-1}e^{-b/x}\,dx = b^{-\mu}a^{\mu-1}\int_{b/a}^{\infty}(t-b/a)^{\mu-1}e^{-t}dt.$$

The change of variable $s = t - a/b$ gives

$$\int_{b/a}^{\infty}(t-b/a)^{\mu-1}e^{-t}dt = e^{-b/a}\int_0^{\infty}s^{\mu-1}e^{-s}ds,$$

and the last integral is $\Gamma(\mu)$.