PROOF OF FORMULA 3.472.4

$$\int_0^\infty \exp\left[-\frac{1}{2a}\left(x^2+\frac{1}{x^2}\right)\right]\frac{dx}{x^4} = \sqrt{\frac{a\pi}{2}}(1+a)e^{-1/a}$$

Formula 3.325 states that

$$\int_0^\infty e^{-ax^2-b/x^2}\,dx = \frac{1}{2}\sqrt{\frac{\pi}{a}}e^{-2\sqrt{ab}}.$$

Differentiate twice with respect to the parameter a to obtain

$$\int_0^\infty e^{-ax^2-b/x^2}\,\frac{dx}{x^4} = \frac{\sqrt{\pi}}{2a}e^{-2\sqrt{ab}}\left(\sqrt{b}+\frac{1}{2\sqrt{a}}\right).$$

The result follows by putting $a=b$ and then replacing a by $1/2a$.