PROOF OF FORMULA 3.472.5

$$\int_0^\infty x^{-n-\frac{1}{2}} e^{-bx-a/x} \, dx = (-1)^n \sqrt{\frac{\pi}{b}} \left(\frac{\partial}{\partial a} \right)^n e^{-2\sqrt{ab}}$$

Formula 3.325 states that

$$\int_0^\infty \exp\left(-\frac{a}{x^2} - bx^2 \right) \, dx = \frac{\sqrt{\pi}}{2\sqrt{b}} e^{-2\sqrt{ab}}.$$

The change of variables $t = x^2$ gives

$$\int_0^\infty \exp\left(-\frac{a}{t} - bt \right) \frac{dt}{\sqrt{t}} = \frac{\sqrt{\pi}}{2\sqrt{b}} e^{-2\sqrt{ab}}.$$

Now observe that every differentiation with respect to a produces a factor of $1/t$. This gives the result.