PROOF OF FORMULA 3.476.1

$$\int_0^\infty \left[\exp(-\nu x^p) - \exp(-\mu x^p) \right] \frac{dx}{x} = \frac{1}{p} \ln \left(\frac{\mu}{\nu} \right)$$

Let $t = \nu x^p$ to obtain

$$\int_0^\infty \left[\exp(-\nu x^p) - \exp(-\mu x^p) \right] \frac{dx}{x} = \frac{1}{p} \int_0^\infty \left[e^{-t} - e^{-\mu t/\nu} \right] \frac{dt}{t}.$$

Use the representation

$$\gamma = -\int_0^\infty \left(e^{-t} - \frac{1}{1+t} \right) \frac{dt}{t}$$

and the change opf variables $s = \mu t/\nu$ to write

$$\int_0^\infty \left[\exp(-\nu x^p) - \exp(-\mu x^p) \right] \frac{dx}{x} = -\frac{\gamma}{p} - \frac{1}{p} \int_0^\infty \left(e^{-s} - \frac{1}{1+\nu s/\mu} \right) \frac{ds}{s}.$$

Simplifying it gives

$$\int_0^\infty \left[\exp(-\nu x^p) - \exp(-\mu x^p) \right] \frac{dx}{x} = -\frac{1}{p} \int_0^\infty \left(\frac{\nu}{\mu + \nu s} - \frac{1}{1+s} \right) ds.$$

These are elementary integrals that produce the result.