PROOF OF FORMULA 3.481.1

$$\int_{-\infty}^{\infty} xe^x \exp(-\mu e^x)\, dx = -\frac{\gamma + \ln \mu}{\mu}$$

Let $t = \mu e^x$ to obtain

$$\int_{-\infty}^{\infty} xe^x \exp(-\mu e^x)\, dx = \frac{1}{\mu}\left(\int_0^{\infty} e^{-t} \ln t\, dt - \ln \mu \int_0^{\infty} e^{-t}\, dt\right).$$

The first integral is $\Gamma'(1) = -\gamma$ and the second one is 1. This gives the result.