PROOF OF FORMULA 3.511.4

$$\int_0^\infty \frac{\cosh ax}{\cosh bx}\, dx = \frac{\pi}{2b} \sec\left(\frac{\pi a}{2b}\right)$$

Write the integral as

$$\int_0^\infty \frac{\cosh ax}{\cosh bx}\, dx = \int_0^\infty \frac{e^{(a-b)x} + e^{-(a+b)x}}{1+e^{-2bx}}\, dx.$$

Now expand the integrand as a geometric series to obtain

$$\begin{aligned}
\int_0^\infty \frac{\cosh ax}{\cosh bx}\, dx &= \sum_{k=0}^\infty (-1)^k \int_0^\infty \left(e^{-(b-a+2bk)x} + e^{-(a+b+2bk)x} \right) \\
&= \sum_{k=0}^\infty (-1)^k \left(\frac{1}{b-a+2bk} + \frac{1}{b+a+2bk} \right) \\
&= \frac{2}{b} \sum_{k=0}^\infty \frac{(-1)^k (2k+1)}{(2k+1)^2 - (a/b)^2}.
\end{aligned}$$

The result now follows from the representation

$$\sec\left(\frac{\pi u}{2}\right) = \frac{4}{\pi} \sum_{k=0}^\infty (-1)^k \frac{(2k+1)}{(2k+1)^2 - u^2}.$$