

**PROOF OF FORMULA 3.511.5**

$$\int_0^{\infty} \frac{\sinh ax \cosh bx}{\sinh cx} dx = \frac{\pi}{2c} \left( \frac{\sin \frac{\pi a}{c}}{\cos \frac{\pi a}{c} + \cos \frac{\pi b}{c}} \right)$$

Express the integrand in exponential form to obtain

$$\int_0^{\infty} \frac{\sinh ax \cosh bx}{\sinh cx} dx = \frac{1}{2} \int_0^{\infty} \frac{e^{-cx}(e^{ax} - e^{-ax})(e^{bx} + e^{-bx})}{1 - e^{-2cx}} dx.$$

The change of variables  $t = e^{-2cx}$  gives

$$\int_0^{\infty} \frac{\sinh ax \cosh bx}{\sinh cx} dx = \frac{1}{4c} \int_0^1 \frac{t^{-A-B-1/2} + t^{-A+B-1/2} - t^{A-B-1/2} - t^{A+B-1/2}}{1-t} dt$$

where  $A = \frac{a}{2c}$  and  $B = \frac{b}{2c}$ . Using the formula

$$\int_0^1 \frac{1-x^{a-1}}{1-x} dx = \psi(a) + \gamma$$

in entry 3.265, it follows that

$$\int_0^{\infty} \frac{\sinh ax \cosh bx}{\sinh cx} dx = \frac{1}{4c} \left( \psi\left(\frac{1}{2} + A - B\right) - \psi\left(\frac{1}{2} - A + B\right) + \psi\left(\frac{1}{2} + A + B\right) - \psi\left(\frac{1}{2} - A - B\right) \right).$$

The result now follows from the identity

$$\psi\left(\frac{1}{2} + z\right) - \psi\left(\frac{1}{2} - z\right) = \pi \tan \pi z.$$