# PROOF OF FORMULA 3.511.7

$$\int_0^\infty \frac{\sinh ax \sinh bx}{\cosh cx}\, dx = \frac{\pi \, \sin \frac{\pi a}{2c} \sin \frac{\pi b}{2c}}{c \left(\cos \frac{\pi a}{c} + \cos \frac{\pi b}{c}\right)}$$

Write the integrand in exponential form and let $t = e^{-2cx}$ to obtain

$$\int_0^\infty \frac{\sinh ax \sinh bx}{\cosh cx}\, dx = \frac{1}{4c} \int_0^1 \frac{t^{\frac{1}{2}-A-B} - t^{\frac{1}{2}+A-B} - t^{\frac{1}{2}-A+B} + t^{\frac{1}{2}+A+B}}{1+t} \, \frac{dt}{t},$$

where $A = a/2c$ and $B = b/2c$. This can be expressed in terms of the incomplete beta function

$$\beta(a) = \int_0^1 \frac{t^{a-1}\, dt}{1+t}$$

as

$$\int_0^\infty \frac{\sinh ax \sinh bx}{\cosh cx}\, dx = \tfrac{1}{4c} \left[ \beta\left(\tfrac{1}{2} - \tfrac{a+b}{2c}\right) - \beta\left(\tfrac{1}{2} + \tfrac{a-b}{2c}\right) - \beta\left(\tfrac{1}{2} - \tfrac{a-b}{2c}\right) + \beta\left(\tfrac{1}{2} + \tfrac{a+b}{2c}\right)\right].$$

The result now follows from the rule

$$\beta\left(\frac{1}{2} - \frac{z}{2}\right) + \beta\left(\frac{1}{2} + \frac{z}{2}\right) = \pi \sec\left(\frac{\pi z}{2}\right).$$