PROOF OF FORMULA 3.511.8

$$\int_0^\infty \frac{dx}{\cosh(x^2)} = \sqrt{\pi} \sum_{k=0}^\infty \frac{(-1)^k}{\sqrt{2k+1}}$$

Write the integral as

$$\int_0^\infty \frac{dt}{\cosh(t^2)} = 2 \int_0^\infty \frac{e^{-x^2}\, dx}{1+e^{-2x^2}}$$

and expand the integrand as a power series to obtain

$$\int_0^\infty \frac{e^{-x^2}\, dx}{1+e^{-2x^2}} = \sum_{k=0}^\infty (-1)^k \int_0^\infty e^{-(2k+1)x^2}\, dx.$$

The change of variables $x = t/\sqrt{2k+1}$ gives

$$\int_0^\infty \frac{e^{-x^2}\, dx}{1+e^{-2x^2}} = \sum_{k=0}^\infty \frac{(-1)^k}{\sqrt{2k+1}} \int_0^\infty e^{-t^2}\, dt.$$

This last integral has value $\sqrt{\pi}/2$. The formula has been established.