PROOF OF FORMULA 3.513.2

$$\int_0^\infty \frac{dx}{a + b\cosh x} = \begin{cases} \frac{2}{\sqrt{b^2-a^2}} \tan^{-1} \sqrt{\frac{b-a}{b+a}} & \text{if } b^2 > a^2 \\ \\ \frac{1}{\sqrt{a^2-b^2}} \ln \left[\frac{a+b+\sqrt{a^2-b^2}}{a+b-\sqrt{a^2-b^2}} \right] & \text{if } b^2 < a^2 \end{cases}$$

Write the hyperbolic cosine as exponentials and let $t = e^{-x}$ to obtain

$$\int_0^\infty \frac{dx}{a + b\cosh x} = 2 \int_0^1 \frac{dt}{bt^2 + 2at + b}.$$

Complete squares to obtain

$$\int_0^\infty \frac{dx}{a + b\cosh x} = \frac{2}{b} \int_c^{1+c} \frac{ds}{s^2 + 1 - c^2},$$

with $c = a/b$. The computation is now divided according to whether $c^2 > 1$ or not. The details are elementary.