

PROOF OF FORMULA 3.522.10

$$\int_0^{\infty} \frac{x dx}{(1+x^2) \cosh \frac{\pi x}{4}} = \frac{1}{\sqrt{2}} \left(\pi - 2 \ln(\sqrt{2} + 1) \right)$$

This is entry 3.522.3 with $a = \pi/4$ and $b = 1$. Therefore

$$\int_0^{\infty} \frac{x dx}{(1+x^2) \cosh \frac{\pi x}{4}} = 4 \sum_{k=1}^{\infty} \frac{(-1)^{k-1}}{4k-1}.$$

To evaluate the series integrate

$$\sum_{k=1}^{\infty} (-1)^k x^{4k-2} = \frac{x^2}{1+x^4}$$

to obtain

$$\sum_{k=1}^{\infty} \frac{(-1)^{k-1}}{4k-1} = \int_0^1 \frac{x^2 dx}{1+x^4}.$$

The factorization

$$1+x^4 = (x^2 - \sqrt{2}x + 1)(x^2 + \sqrt{2}x + 1)$$

gives the integral by partial fractions.