PROOF OF FORMULA 3.522.2

$$\int_0^\infty \frac{x\,dx}{(b^2+x^2)\,\sinh \pi x} = \frac{1}{2b} - \beta(b+1)$$

This is the special case $a = \pi$ in entry 3.522.1 that yields

$$\int_0^\infty \frac{x\,dx}{(b^2+x^2)\,\sinh \pi x} = \frac{1}{2b} + \sum_{k=1}^{\infty} \frac{(-1)^k}{k+b}.$$

The result now follows from the expansion

$$\beta(x) = \sum_{k=0}^{\infty} \frac{(-1)^k}{k+x}$$

given as entry 8.372.