

PROOF OF FORMULA 3.522.3

$$\int_0^{\infty} \frac{dx}{(b^2 + x^2) \cosh ax} = \frac{2\pi}{b} \sum_{k=1}^{\infty} \frac{(-1)^{k-1}}{2ab + (2k-1)\pi}$$

The change of variables $x = bt$ shows that it suffices to assume $b = 1$. Extend the integrand f to the whole real line using its symmetry. The integrand has poles in the upper half plane at $z = i$ and $z = \frac{(2k+1)\pi i}{2a}$, $k \geq 0$. Assume first that the poles are simple.

Integrate over a semi-circle centered at the origin and radius R . The residues are

$$\begin{aligned} \operatorname{Res}(f; i) &= \frac{1}{2i \cosh(ia)} = \frac{1}{2i \cos a}, \\ \operatorname{Res}\left(f; \frac{(2k-1)\pi i}{2a}\right) &= \frac{(-1)^{k-1} 4ia}{4a^2 - \pi^2(2k-1)^2}. \end{aligned}$$

The residue theorem and a partial fraction decomposition give the stated value of the integral.