

**PROOF OF FORMULA 3.522.5**

$$\int_0^{\infty} \frac{x dx}{(1+x^2) \sinh \pi x} = \ln 2 - \frac{1}{2}$$

This is the special case  $b = 1$  of entry 3.522.2 that gives

$$\int_0^{\infty} \frac{x dx}{(1+x^2) \sinh \pi x} = \frac{1}{2} - \beta(2),$$

where

$$\beta(2) = \sum_{k=0}^{\infty} \frac{(-1)^k}{k+2} = \sum_{k=2}^{\infty} \frac{(-1)^k}{k} = -\ln 2 + 1.$$

This gives the result.