

PROOF OF FORMULA 3.522.7

$$\int_0^{\infty} \frac{x dx}{(1+x^2) \sinh \frac{\pi x}{2}} = \frac{\pi}{2} - 1$$

This is the special case $a = \pi/2$ and $b = 1$ of entry 3.522.1. It yields

$$\int_0^{\infty} \frac{x dx}{(1+x^2) \sinh \frac{\pi x}{2}} = 1 + 2 \sum_{k=1}^{\infty} \frac{(-1)^k}{2k+1}.$$

The result follows from the expansion

$$\sum_{k=0}^{\infty} \frac{(-1)^k}{2k+1} = \frac{\pi}{4}.$$