

PROOF OF FORMULA 3.523.12

$$\int_0^{\infty} \frac{dx}{\sqrt{x} \cosh x} = 2\sqrt{\pi} \sum_{k=0}^{\infty} \frac{(-1)^k}{\sqrt{2k+1}}$$

Let $x = t^2$ to obtain

$$\int_0^{\infty} \frac{dx}{\sqrt{x} \cosh x} = 2 \int_0^{\infty} \frac{dt}{\cosh(t^2)}.$$

The value of this last integral appears in formula 3.511.8. It is established by expanding the integrand in

$$\int_0^{\infty} \frac{dt}{\cosh(t^2)} = 2 \int_0^{\infty} \frac{e^{-x^2} dx}{1 + e^{-2x^2}}$$

as a power series to obtain

$$\int_0^{\infty} \frac{e^{-x^2} dx}{1 + e^{-2x^2}} = \sum_{k=0}^{\infty} (-1)^k \int_0^{\infty} e^{-(2k+1)x^2} dx.$$

The change of variables $x = t/\sqrt{2k+1}$ gives

$$\int_0^{\infty} \frac{e^{-x^2} dx}{1 + e^{-2x^2}} = \sum_{k=0}^{\infty} \frac{(-1)^k}{\sqrt{2k+1}} \int_0^{\infty} e^{-t^2} dt.$$

This last integral has value $\sqrt{\pi}/2$. The formula has been established.