

PROOF OF FORMULA 3.523.2

$$\int_0^\infty \frac{x^{2n-1} dx}{\sinh ax} = \frac{2^{2n} - 1}{2n} \left(\frac{\pi}{a}\right)^{2n} |B_{2n}|$$

The value $b = 2n$ in 3.523.1:

$$\int_0^\infty \frac{x^{b-1} dx}{\sinh ax} = \frac{2^b - 1}{2^{b-1} a^b} \Gamma(b) \zeta(b),$$

gives

$$\int_0^\infty \frac{x^{2n-1} dx}{\sinh ax} = \frac{2^{2n} - 1}{2^{2n-1} a^{2n}} \Gamma(2n) \zeta(2n).$$

The result now follows from the evaluation

$$\zeta(2n) = \frac{2^{2n-1} \pi^{2n} |B_{2n}|}{(2n)!}.$$