

**PROOF OF FORMULA 3.523.3**

$$\int_0^{\infty} \frac{x^{b-1} dx}{\cosh ax} = \frac{2\Gamma(b)}{(2a)^b} \sum_{k=0}^{\infty} (-1)^k \left( \frac{2}{2k+1} \right)^b$$

The change of variable  $t = ax$  gives

$$\int_0^{\infty} \frac{x^{b-1} dx}{\cosh ax} = \frac{2}{a^b} \int_0^{\infty} \frac{t^{b-1} e^{-t} dt}{1 + e^{-2t}}.$$

Expand the integrand in series to obtain

$$\int_0^{\infty} \frac{t^{b-1} e^{-t} dt}{1 + e^{-2t}} = \sum_{k=0}^{\infty} (-1)^k \int_0^{\infty} t^{b-1} e^{-(2k+1)t} dt.$$

The change of variables  $u = (2k+1)t$  gives the result after recognizing the value

$$\int_0^{\infty} u^{b-1} e^{-u} du = \Gamma(b).$$