

PROOF OF FORMULA 3.523.4

$$\int_0^\infty \frac{x^{2n} dx}{\cosh ax} = \left(\frac{\pi}{2a}\right)^{2n+1} |E_{2n}|$$

Entry 3.523.3 states that

$$\int_0^\infty \frac{x^{b-1} dx}{\cosh ax} = \frac{2\Gamma(b)}{(2a)^b} \sum_{k=0}^\infty (-1)^k \left(\frac{2}{2k+1}\right)^b.$$

The special case $b = 2n + 1$ gives

$$\int_0^\infty \frac{x^{2n} dx}{\cosh ax} = \frac{2\Gamma(b)}{(2a)^{2n+1}} \sum_{k=0}^\infty (-1)^k \left(\frac{2}{2k+1}\right)^{2n+1}.$$

The result follows from the classical representation of the Euler numbers

$$\sum_{k=0}^\infty \frac{(-1)^k}{(2k+1)^{2n+1}} = \frac{\pi^{2n+1} |E_{2n}|}{(2n)! 2^{2n+2}}.$$