

PROOF OF FORMULA 3.523.6

$$\int_0^{\infty} \frac{x^3 dx}{\sinh x} = \frac{\pi^4}{8}$$

Formula 3.523.2 states that

$$\int_0^{\infty} \frac{x^{2n-1} dx}{\sinh ax} = \frac{2^{2n} - 1}{2n} \left(\frac{\pi}{a}\right)^{2n} |B_{2n}|.$$

The special case $n = 2$ and $a = 1$, using the value $B_4 = -1/30$ give the result.