PROOF OF FORMULA 3.524.9

$$\int_0^\infty x^2 \frac{\sinh ax}{\sinh bx}\,dx = \frac{\pi^3}{4b^3}\sin\left(\frac{\pi a}{2b}\right)\sec^3\left(\frac{\pi a}{2b}\right)$$

Entry 3.524.2 states that

$$\int_0^\infty x^2 \frac{\sinh ax}{\sinh bx}\,dx = \frac{\pi}{2b}\left(\frac{d}{da}\right)^2 \tan\left(\frac{\pi a}{2b}\right).$$

Now simply evaluate the second derivative to produce the result.