PROOF OF FORMULA 3.527.10

$$\int_0^\infty x^{2m} \frac{\cosh ax}{\sinh^2 ax}\, dx = \frac{(2^{2m}-1)}{a} \left(\frac{\pi}{a}\right)^{2m} |B_{2m}|$$

Entry 3.527.9 states that

$$\int_0^\infty x^{\mu-1} \frac{\cosh ax}{\sinh^2 ax}\, dx = \frac{2\Gamma(\mu)\zeta(\mu-1)}{a^\mu} \left(1-2^{1-\mu}\right).$$

Put $\mu = 2m+1$ and use the relation

$$\zeta(2m) = \frac{2^{2m-1}\pi^{2m}}{(2m)!}|B_{2m}|.$$