

PROOF OF FORMULA 3.527.12

$$\int_{-\infty}^{\infty} \frac{x^2 dx}{\sinh^2 x} = \frac{\pi^2}{3}$$

Entry 3.527.1 states that

$$\int_0^{\infty} \frac{x^{\mu-1} dx}{\sinh^2(ax)} = \frac{4\Gamma(\mu)\zeta(\mu-1)}{(2a)^\mu}.$$

Put $a = 1$ and $\mu = 3$ to obtain

$$\int_0^{\infty} \frac{x^2 dx}{\sinh^2 x} = \frac{4\Gamma(3)\zeta(2)}{2^3}.$$

This is the result.