## PROOF OF FORMULA 3.527.15

$$\int_0^\infty \frac{\tanh(x/2)}{\cosh x}\,dx = \ln 2$$

Start with

$$\int_0^\infty \frac{\tanh(x/2)}{\cosh x}\,dx = 2\int_0^\infty \frac{e^x-1}{e^x+1}\,\frac{e^x}{e^{2x}+1}\,dx,$$

and let $t = e^{-x}$ to obtain

$$2\int_0^\infty \frac{e^x-1}{e^x+1}\,\frac{e^x}{e^{2x}+1}\,dx = 2\int_0^1 \frac{1-t}{(1+t)(1+t^2)}\,dt.$$

Expanding by partial fractions it follows that

$$2\int_0^1 \frac{1-t}{(1+t)(1+t^2)}\,dt = 2\int_0^1 \frac{dt}{1+t} - 2\int_0^1 \frac{t\,dt}{1+t^2} = \int_0^1 \frac{dt}{1+t} = \ln 2.$$