

PROOF OF FORMULA 3.527.7

$$\int_0^{\infty} \frac{x \sinh ax}{\cosh^2 ax} dx = \frac{\pi}{2a^2}$$

Formula 3.527.6 states that

$$\int_0^{\infty} \frac{x^{\mu-1} \sinh ax}{\cosh^2 ax} dx = \frac{2\Gamma(\mu)}{a^{\mu}} \sum_{k=0}^{\infty} \frac{(-1)^k}{(2k+1)^{\mu-1}}.$$

In the special case $\mu = 2$ this gives

$$\int_0^{\infty} \frac{x \sinh ax}{\cosh^2 ax} dx = \frac{2\Gamma(2)}{a^2} \sum_{k=0}^{\infty} \frac{(-1)^k}{2k+1}.$$

The value

$$\sum_{k=0}^{\infty} \frac{(-1)^k}{2k+1} = \frac{\pi}{4}$$

gives the result.