

PROOF OF FORMULA 3.527.8

$$\int_0^\infty \frac{x^{2m+1} \sinh ax}{\cosh^2 ax} dx = \frac{2m+1}{a} \left(\frac{\pi}{2a}\right)^{2m+1} |E_{2m}|$$

Entry 3.527.6 states that

$$\int_0^\infty \frac{x^{\mu-1} \sinh ax}{\cosh^2 ax} dx = \frac{2\Gamma(\mu)}{a^\mu} \sum_{k=0}^\infty \frac{(-1)^k}{(2k+1)^{\mu-1}}.$$

The result follows by putting $\mu = 2m + 2$ and the formula

$$\sum_{k=0}^\infty \frac{(-1)^k}{(2k+1)^{2m+1}} = \frac{\pi^{2m+1} |E_{2m}|}{(2m)! 2^{2m+2}}.$$

Here E_m is the Euler number.