PROOF OF FORMULA 3.542.1

$$\int_0^\infty e^{-\mu x} \left(\cosh \beta x - 1\right)^\nu \, dx = \frac{1}{2^\nu \beta} B\left(\frac{\mu}{\beta} - \nu, 2\nu + 1\right)$$

The change $t = \beta x$ shows that the entry is equivalent to

$$\int_0^\infty e^{-ct} \left(\cosh t - 1\right)^\nu \, dt = \frac{1}{2^\nu} B\left(c - \nu, 2\nu + 1\right).$$

To prove this, let $y = e^{-t}$ to obtain

$$\int_0^\infty e^{-ct} \left(\cosh t - 1\right)^\nu \, dt = (-1)^{\nu-1} \int_0^1 y^{c-\nu-1}(1 - y)^{2\nu} \, dy.$$

This gives the result.